# FOCS Homework 16

You may edit your answers into this file, or add a separate file in the same directory.

If you add a separate file, please include the following at the top:

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Student Name: Frankly Olin [change to your name]

Check one:

[ ] I completed this assignment without assistance or external resources.

[ ] I completed this assignment with assistance from \_\_\_

and/or using these external resources: \_\_\_

```

## I. (Undirected) Graphs

A \*\*graph\*\* is a set of points (called \*\*nodes\*\* or \*\*vertices\*\*), connected pair-wise by lines (called \*\*edges\*\*).

Formally, a graph is an ordered pair \*G\* = (\*V\*, \*E\*): \*V\* is a set of vertices; E is a set of edges; and an edge is a set of the nodes that are its ends.

\*Example\*:

![](images/graph-example.svg)

\*G\* = ({1, 2, 3}, {{1, 2}, {1, 3}})

### 1. Match the diagrams to the graph structures.

![](images/graphs.svg)

(i) \*G\* = ({1, 2, 3, 4}, {{1, 2}, {1, 4}, {3, 4}}) **b**

(ii) \*G\* = ({1, 2, 3, 4}, {{1, 2}, {1, 4}, {2, 3}, {3, 4}}) **a**

(iii) \*G\* = ({1, 2, 3, 4, 5}, {{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 5}}) **d**

(iv) \*G\* = ({1, 2, 3, 4}, {{1, 2}, {1, 4}, {1, 3}}) **c**

### 2. Cycles

Two vertices are \*\*adjacent\*\* if there's an edge between them.

A \*\*cycle\*\* is a sequence of adjacent vertices that begin and end with the same vertex.

Which of (1a-d) contains a cycle? For those graphs that contain a cycle, what is the cycle?

**1a contains a cycle 1-2-3-4. 1d contains a cycle 1-2-5.**

### 3. Draw a diagram for this graph.

\*G\* = ({1, 2, 3, 4, 5}, {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {1, 5}})

2

1

5

3

4

### 4. Write the graph (V, E) for this diagram.

![](images/graph.svg)

This is a single graph. It is not a \*\*connected graph\*\*.

**\*G\* = ({1, 2, 3, 4, 5, 6}, {{1, 2}, {1,3}, {1,4}, {2, 3}, {3, 4}, {3,5}, {4, 5}})**

## II. Directed Graphs

A \*\*directed graph\*\* (â€œdigraphâ€) is a set of vertices, together with arrows that begin and end on a vertex.

Formally, \*G\* = (\*V\*, \*A\*), where \*A\* is a set of tuples $(V\_i, V\_j)$.

### 5. Match the diagrams to the graphs.

![](images/digraphs.svg)

(i) \*G\* = ({1, 2, 3, 4}, {(1, 2), (1, 4), (3, 2), (3, 4)}) **a**

(ii) \*G\* = ({1, 2, 3, 4}, {(1, 2), (1, 4), (4, 3)})**c**

(iii) \*G\* = ({1, 2, 3, 4}, {(1, 2), (4, 1), (2, 3), (3, 4)})**b**

### 6. Directed acyclic graphs

A cycle in a directed graph is a sequence of vertices, that starts and ends with the same vertex, where for each pair of consecutive vertices in the sequence there's an arrow from the first to the second.

A \*\*directed acycle graph\*\* (DAG) is a digraph that doesn't contain cycles.

Which of (5a-c) are DAGs?

**5a and 5c are DAGs.**

### 7. Draw a diagram for this graph.

2

1

3

4

\*G\* = ({1, 2, 3, 4}, {(1, 2), (2, 3), (3, 4), (4, 1), (1, 3)})

Is this graph a DAG?

**No. 1-2-3-4-1 is a cycle.**

### 6. Write the graph (V, A) for this diagram.

![](images/digraph.svg)

**\*G\* = ({1, 2, 3, 4}, {(1, 2), (2, 4), (3, 1), (3, 2), (3, 4)})**

### 7. Adjacency matrices

An \*\*adjacency matrix\*\* is a square matrix \*A\* such that $A\_{i,j}$ = 1 iff there is an arrow from vertex \*i\* to vertex \*j\*.

Write the adjacency matrix for the graph in (5a).

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **j=1** | **j=2** | **j=3** | **j=4** |
| **i=1** | **0** | **1** | **0** | **1** |
| **i=2** | **0** | **0** | **0** | **0** |
| **i=3** | **0** | **1** | **0** | **1** |
| **i=4** | **0** | **0** | **0** | **0** |

## III. (Optional) Reading

One of:

\* Cormen \*et al.\* Section 6 â€œGraph Algorithmsâ€, Chapters 22-24.

\* Equivalent material in your favorite data structures text: graphs, Hamiltonian paths, algorithms to compute spanning trees, shortest paths.

\* Wikipedia: [Graph](https://en.wikipedia.org/wiki/Graph\_(discrete\_mathematics)), [graph data type](https://en.wikipedia.org/wiki/Graph\_(abstract\_data\_type)), [directed graph](https://en.wikipedia.org/wiki/Directed\_graph), [adjacency matrix](https://en.wikipedia.org/wiki/Adjacency\_matrix), [Hamiltonian path](https://en.wikipedia.org/wiki/Hamiltonian\_path)